Mathematics III Frameworks Student Edition

Unit 3 Exploring Exponentia (Exponents and Logarithms)

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Exploring Exponentia
(Exponents and Logarithms)
Student Edition

INTRODUCTION:

In Math 2, students learned about exponential functions, including the natural exponential function, *e*. Additionally, throughout the previous grades and units, students have explored basic functions, inverse functions, and the impact of different transformations on the graphs of functions. This unit builds on students' understandings of inverse functions, exponential functions, and transformations of functions. Before delving into logarithmic functions, it is important for students to understand *nth* roots and rational exponents, so the study of these topics is motivated in the first tasks. Review of the essential prerequisites is used as a way to motivate the study of logarithms, primarily common and natural logarithms. This unit is not concerned with the properties of logarithms or with solving complex logarithmic equations. Rather, it focuses on building student conceptual understandings of logarithms.

ENDURING UNDERSTANDINGS:

- *Nth* roots are inverses of power functions. Understanding the properties of power functions and how inverses behave explains the properties of *nth* roots.
- Real-life situations are rarely modeled accurately using discrete data. It is often necessary to introduce rational exponents to model and make sense of a situation.
- Computing with rational exponents is no different from computing with integral exponents.
- Logarithmic functions are inverses of exponential functions. Understanding the properties of exponential functions and how inverses behave explains the properties and graphs of logarithms.
- Exponential and logarithmic functions behave the same as other functions with respect to graphical transformations.
- Two special logarithmic functions are common logarithms and natural logarithms. These special functions occur often in nature.

KEY STANDARDS ADDRESSED:

MM3A2. Students will explore logarithmic functions as inverses of exponential functions.

a. Define and understand the properties of nth roots.
b. Extend the properties of exponents to include rational exponents.
c. Define logarithmic functions as inverses of exponential functions.

e. Investigate and explain characteristics of exponential and logarithmic functions including domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, and rate of change.
f. Graph functions as transformations of $f(x) = a^x$, $f(x) = log_a x$, $f(x) = e^x$, and $f(x) = ln\ x$.
g. Explore real phenomena related to exponential and logarithmic functions including half-time and doubling time.

RELATED STANDARDS ADDRESSED:

MM3A1. Students will analyze graphs of polynomial functions of higher degree.
d. Investigate and explain characteristics of polynomial functions, including domain and range, intercepts, zeros, relative and absolute extrema, intervals of increase and decrease, and end behavior.

MM3A3. Students will solve a variety of equations and inequalities.
b. Solve polynomial, exponential, and logarithmic equations analytically, graphically, and using appropriate technology.
d. Solve a variety of types of equations by appropriate means choosing among mental calculation, pencil and paper, or appropriate technology.

MM1P1. Students will solve problems (using appropriate technology).
a. Build new mathematical knowledge through problem solving.
b. Solve problems that arise in mathematics and in other contexts.
c. Apply and adapt a variety of appropriate strategies to solve problems.
d. Monitor and reflect on the process of mathematical problem solving.

MM1P2. Students will reason and evaluate mathematical arguments.
a. Recognize reasoning and proof as fundamental aspects of mathematics.
b. Make and investigate mathematical conjectures.
c. Develop and evaluate mathematical arguments and proofs.
d. Select and use various types of reasoning and methods of proof.

MM1P3. Students will communicate mathematically.
a. Organize and consolidate their mathematical thinking through communication.
b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
c. Analyze and evaluate the mathematical thinking and strategies of others.
d. Use the language of mathematics to express mathematical ideas precisely.

MM1P4. Students will make connections among mathematical ideas and to other disciplines.
a. Recognize and use connections among mathematical ideas.
b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
c. Recognize and apply mathematics in contexts outside of mathematics.

MM1P5. Students will represent mathematics in multiple ways.

a. Create and use representations to organize, record, and communicate mathematical ideas.
b. Select, apply, and translate among mathematical representations to solve problems.
c. Use representations to model and interpret physical, social, and mathematical phenomena.

Unit Overview:

The launching task reviews a number of ideas students learned in Math 2 that are essential understandings for this unit. Specifically, knowledge and facility with the content in MM2A5 is necessary for completing this task. Students are required to recall knowledge about the relationships between functions and their inverses. Students must also recall the surface area and volume of spheres, another topic from Math 2. Beginning in this task and continuing throughout the unit is an emphasis on multiple representations, i.e. using tables, graphs, and equations.

The second task continues reviewing content, primarily exponential functions, and motivates the need for determining the inverse of an exponential function. Also addressed is the need for rational exponents in working with exponential equations. The properties of exponents are reviewed and students practice working with rational exponents. Solving exponential equations graphically is stressed in this task. Students also determine half-lifes and doubling time in the task.

Task 3 builds on students' understandings of exponential functions and inverses to develop the idea of a logarithm. Through paper folding and examining graphs, students determine the graph of the inverse of an exponential function, eventually assigning the inverse the name logarithm. Students then work with logarithms and their graphs, including common logarithms, to deepen their understanding and solve exponential problems.

In the fourth task, students model natural phenomena with exponential and logarithmic functions, including natural logarithms. The concept of natural logarithm is developed in context. Throughout the task, students are asked to consider how the equations of natural phenomena are transformations of the basic exponential or logarithm functions.

Finally, the culminating task requires the students to use their knowledge from the unit to answer a variety of questions about naturally occurring phenomena. Many of the questions resemble those asked in the learning tasks, but they generally possess a slightly higher level of complexity. To close the unit, students are asked to use graphs of the functions in this unit to design an alien creature encountered during the unit.

Throughout this unit, including the culminating task, it is assumed that students have access to a graphing utility. Tasks stress knowing the domain and range of functions, partially as a way to help students learn to set their graphing window and partially as a way to judge the validity of their answers. The unit emphasizes the reasonableness of results and models. Students should be encouraged to communicate their reasoning both in writing and in class discussions.

Vocabulary and formulas:

Asymptote: A line or curve that describes the end behavior of the graph. A graph never crosses a vertical asymptote but it may cross a horizontal or oblique asymptote.

Common logarithm: A logarithm with a base of 10. A common logarithm is the power, *a*, such that $10^a = b$. The common logarithm of *x* is written *log x*. For example, $\log 100 = 2$ because $10^2 = 100$.

Compounded continuously: Interest that is, theoretically, computed and added to the balance of an account each instant. The formula is $A = Pe^{rt}$, where A is the ending amount, P is the principal or initial amount, r is the annual interest rate, and t is the time in years.

Compounded interest: A method of computing the interest, after a specified time, and adding the interest to the balance of the account. Interest can be computed as little as once a year to as many times as one would like. The formula is $A = P_0(1 + r/n)^{nt}$, where A is the ending amount, P_o is the initial amount, r is the annual interest rate, n is the number of times compounded per year, and t is the number of years.

Exponential functions: A function of the form $y = a \cdot b^x$ where $a > 0$ and either $0 < b < 1$ or $b > 1$.

Logarithmic functions: A function of the form $y = \log_b x$, with $b \neq 1$ and *b* and *x* both positive. A logarithmic function is the inverse of an exponential function. The inverse of $y = b^x$ is $y = \log_b x$.

Logarithm: The logarithm base *b* of a number *x*, $\log_b x$, is the power to which *b* must be raised to equal *x*.

Natural exponential: Exponential expressions or functions with a base of *e*, i.e. $y = e^x$.

Natural logarithm: A logarithm with a base of *e*. A natural logarithm is the power, *a*, such that $e^a = b$. The natural logarithm of *x* is written *ln x*. For example, $\ln 8 = 2.0794415\ldots$ because $e^{2.0794415\ldots} = 8$.

***Nth* roots:** The number that must be multiplied by itself *n* times to equal a given value. The *nth* root can be notated with radicals and indices or with rational exponents, i.e. $x^{1/3}$ means the cube root of *x*.

Rational exponents: For $a > 0$, and integers *m* and *n*, with $n > 0$, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$; $a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$.

Transcendental numbers: Real numbers that are not algebraic. That is, they are not solutions to polynomial equations with integer coefficients. Two examples are *e* and π.

THE PLANET OF EXPONENTIA LEARNING TASK:

A new solar system was discovered far from the Milky Way in 1999. One of the planets in the system, Exponentia, has a number of unique characteristics. Scientists noticed that the radius of the planet has been increasing 500 meters each year.

When NASA scientists first spotted Exponentia, its diameter was approximately 40 km.

1. Make a table that lists the diameter and surface area of the planet from the years 1999 to 2009. (Leave your surface area answers in terms of π.)

	Diameter (in km)	Surface Area (in km^2)
1999	40	
2000		
2001		
2002		
2003		
2004		
2005		
2006		
2007		
2008		
2009		

a. Write a function rule that expresses the relationship between the radius of the planet and its surface area. What does it mean for the surface area to be a function of the radius (or diameter)? (Make sure you use proper function notation.)

b. Interchange the columns and create a second table so that surface area is the independent variable and diameter is the dependent.

	Surface Area (in km^2)	Diameter (in km)
1999		40
2000		
2001		
2002		
2003		
2004		
2005		
2006		
2007		
2008		
2009		

c. Graph the data from the first table. (Unless you are graphing on a calculator or computer, graphing every other point is sufficient.) How would a graph of the data from the second table look? How do you know?

d. If we wrote a rule (equation) for the new relationship in part (b), how would the new rule be related to the original? That is, how are the two rules related to each other? How do you know?

e. Using algebra, write a rule for the data in the second table. (Hint: We want an equation for the radius in terms of the surface area.)

f. What are the domain and range of the function in part (a)? What are the domain and range for the new relation in part (e)? What are the restrictions on the domain and range due to the context of the problem? Why are there restrictions?

Function in Part (a): Domain ______________ Range ______________________

Restrictions due to context:

Relation in Part (e): Domain ______________ Range ______________________

Restrictions due to context:

g. Is the new relation in part (e) also a function? How do you know? Explain two ways: using the graph of the original function in part (a) and using the graph of the unrestricted relation in part (e). (You may need to graph your equations on your graphing calculator or computer.)

2. Now let’s consider how the volume of the planet changes.

   a. Write a function rule that expresses the relationship between the radius of the planet and its volume.

   b. Graph the function from part (a) over the interval $-10 \le r \le 10$. What part, if any, of this graph makes sense in the context of Exponentia’s volume? Explain.

   c. Using your exploration from part 1, consider the following.

      i. If you wrote an equation with volume as the independent variable and the radius as the dependent variable, would this be a function? Explain.

ii. Describe how the graph of the new equation would look. Sketch this new graph with your graph in part (b). (Hint: How are the graphs of inverses related to each other?)

d. We want to write a rule for the second relation.

i. Explain how you knew to find the equation in part 1(e).

ii. We can use similar reasoning in for finding the equation in this problem. Let's start by solving for r^3.

iii. Now, to finish solving, we need the inverse of r^3. In number 1, to solve for r^2 by taking the square root of both sides of the equation. Likewise, we take the cube root of both sides of the equation in the above step. Solve for r.

When we take the square root of an expression or number, such as $x^2 = 4$, we must consider both the positive and negative roots of the expression or number, so $x = \pm 2$. Do we need to consider both positive and negative roots when we take cube roots? Why or why not?

3. *nth Roots:*

The cube root of *b* is the number whose cube is b. Likewise, the *nth* root of *b* is the number that when raised to the *nth* power is *b*. For example, the 5th root of 32 is 2 because $2^5 = 32$. We can write the 5th root of 32 as $\sqrt[5]{32}$.

Another notation used to represent taking roots employs exponents. Instead of writing the 5th root of 32 as $\sqrt[5]{32}$, we can write it as $32^{\frac{1}{5}}$. The cube root of 27 can be written as $27^{\frac{1}{3}}$. How do you think we would represent the *nth root* of a number *x?*

For the remainder of (3), consider $f(x) = x^n$ and $g(x) = x^{\frac{1}{n}}$.

a. How do you think the graphs of *f(x)* and *g(x)* are related? Using graph paper and/or a calculator, test your conjecture, letting $n = 1, 2, 3, 4$.

b. Evaluate *f(g(x))* and *g(f(x))*. If you need to, use your examples of $n = 1, 2, 3, 4$ to help you determine these compositions. What do the results tell you about *f(x)* and *g(x)*?

c. Explain how your work in problems 1 and 2 confirms your conclusions in parts (a) and (b) of this problem.

4. Using your investigations above and what you remember from Math 2, write a paragraph summarizing characteristics of inverses of functions, how to find inverses algebraically and graphically, and how to tell if inverses are functions.

HOW LONG DOES IT TAKE? LEARNING TASK:

Before sending astronauts to investigate the new planet of Exponentia, NASA decided to run a number of tests on the astronauts.

1. A specific multi-vitamin is eliminated from an adult male's bloodstream at a rate of about 20% per hour. The vitamin reaches peak level in the bloodstream of 300 milligrams.
    a. How much of the vitamin remains 2 hours after the peak level? 5 hours after the peak level? Make a table of values to record your answers. Write expressions for how you obtain your answers.

Time (hours) since peak	0	1	2	3	4	5
Vitamin concentration in bloodstream (mg)	300					

    b. Using your work from (a), write expressions for each computed value using the original numbers in the problem (300 and 20%). For example, after 2 hours, the amount of vitamin left is [300 * (1 - .2)]* (1 - .2).

    c. Using part (b), write a function for the vitamin level with respect to the number of hours after the peak level, *x*.

    d. How would use the function you wrote in (c) to answer the questions in (a)? Use your function and check to see that you get the same answers as you did in part (a).

    e. After how many hours will there be less than 10 mg of the vitamin remaining in the bloodstream? Explain how you would determine this answer using both a table feature and a graph.

    f. Write an equation that you could solve to determine when the vitamin concentration is exactly 10 mg. Could you use the table feature to solve this equation? Could you use the graph feature? How could you use the intersection feature of your calculator? Solve the problem using one of these methods.

g. How would you solve the equation you wrote in (f) algebraically? What is the first step?

To finish solving the problem algebraically, we must know how to find inverses of exponential functions. This topic will be explored later in this unit.

2. A can of Instant Energy, a 16-ounce energy drink, contains 80 mg of caffeine. Suppose the caffeine in the bloodstream peaks at 80 mg. If ½ of the caffeine has been eliminated from the bloodstream after 5 hours (the half-life of caffeine in the bloodstream is 5 hours), complete the following:

   a. How much caffeine will remain in the bloodstream after 5 hours? 10 hours? 1 hour? 2 hours? Make a table to organize your answers. Explain how you came up with your answers. (Make a conjecture. You can return to your answers later to make any corrections.)

Time (hours) since peak	0	1	2	5	10
Caffeine in bloodstream (mg)	80				

   b. Unlike problem (1), in this problem in which 80% remained after each hour, in this problem, 50% remains after each **5 hours**.

      i. In problem (1), what did the exponent in your equation represent?

      ii. In this problem, our exponent needs to represent the number of 5-hour time periods that elapsed. If you represent 1 hour as 1/5 of a 5-hour time period, how do you represent 2 hours? 3 hours? 10 hours? x hours?

   c. Using your last answer in part (b) as your exponent, write an exponential function to model the amount of caffeine remaining in the blood stream x hours after the peak level.

   d. How would use the function you wrote in (c) to answer the questions in (a)? Use your function and check to see that you get the same answers as you did in part (a). (Be careful with your fractional exponents when entering in the calculator. Use parentheses.) If you need to, draw a line through your original answers in part (a) and list your new answers.

e. Determine the amount of caffeine remaining in the bloodstream 3 hours after the peak level? What about 8 hours after peak level? 20 hours? (Think about how many 5-hour intervals are in the number of hours you're interested in.)

f. Suppose the half-life of caffeine in the bloodstream was 3 hours instead of 5.

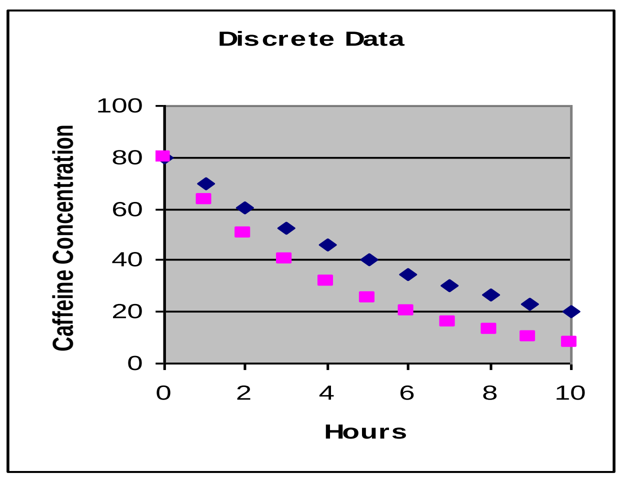
i. Write a function for this new half-life time.

ii. Determine the amount of caffeine in the bloodstream after 1 hour, 2 hours, 5 hours, and 10 hours. (You need to consider how many 3-hour time intervals are used in each time value.)

iii. Which half-life time results in the caffeine being eliminated faster? Explain why this makes sense.

g. Graph both equations (from d and f) on graph paper. How are the graphs similar? Different? What are the intercepts? When are they increasing/decreasing? Are there asymptotes? If so, what are they? Do the graphs intersect? Where?

Note that if we could only use integer exponents; e.g. 1, 2, 3, etc; our graphs would be discontinuous. We would have points (see right), rather than the smooth, continuous curve you graphed above.



It makes sense, in thinking about time, that we need all rational time values, e.g. 1/3 hour, 5/8 hour, etc. This raises the idea of rational exponents, that is, computing values such as $3^{3/4}$ or $(1/2)^{7/3}$.

3. **Rational Exponents.** In previous courses, you learned about different types of numbers and lots of rules of exponents.

a. What are integers? Rational numbers? Which set of numbers is a subset of the other? Explain why this is true.

b. Based on (a), what is the difference between integer exponents and rational exponents?

c. Complete the following exponent rules. (If you don't remember the rules from your previous classes, try some examples to help you.)

For $a > 0$ and $b > 0$, and all values of m and n,

$a^0 =$ ___ $a^1 =$ ____ $a^n =$ __________________________

$(a^m)(a^n) =$ ________ $(a^m)/(a^n) =$ ________ $a^{-n} =$ _______

$(a^m)^n =$ ________ $(ab)^m =$ ___________ $(a/b)^m =$ __________

If $a^m = a^n$, then m ___ n.

The same rules you use for integer exponents also apply to rational exponents.

d. You have previously learned that the nth root of a number x can be represented as $x^{1/n}$.

i. Using your rules of exponents, write another expression for $(x^{1/n})^m$.

ii. Using your rules of exponents, write another expression for $(x^m)^{1/n}$.

iii. What do you notice about the answers in (ii) and (iii)? What does this tell you about rational exponents?

This leads us to the definition of **rational exponents**.

For $a > 0$, and integers m and n, with $n > 0$,

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m; \; a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}.$$

e. Rewrite the following using simplified rational exponents.

i. $\sqrt[7]{x^3}$ ii. $\left(\frac{1}{x}\right)^{-5}$ iii. $\left(\sqrt{x}\right)^6$ iv. $\frac{1}{\sqrt[3]{x^5}}$

f. Simplify each of the following. Show your steps. For example, $27^{2/3} = (27^{1/3})^2 = 3^2 = 9$.

i. $16^{3/4}$ ii. $36^{3/2}$ iii. $81^{7/4}$

Problems involving roots and rational exponents can sometimes be solved by rewriting expressions or by using inverses.

g. Consider the caffeine situation from above. The half-life of caffeine in an adult's bloodstream is 5 hours. How much caffeine is remains in the bloodstream each hour? Our original equation was $\boldsymbol{f(x) = 80(.5)^{x/5}}$**.** Use the rules of rational exponents to rewrite the equation so that the answer to the above question is given in the equation. What percent of the caffeine remains in the bloodstream each hour?

To solve equations such as $x^3 = 27$, we take the cube root of both sides. Alternately, we can raise both sides of the equation to the 1/3 power. That is, we raise both sides of the equation to the power that is the inverse (or reciprocal) of the power in the problem. To solve $x^{3/2} = 27$, we can either square both sides and then take the cube root, we can take the cube root of both sides and then square them, or we can raise both sides to the 2/3 power.

$x^{3/2} = 27 \rightarrow (x^{3/2})^{2/3} = 27^{2/3} \rightarrow x = (27^{1/3})^2 \rightarrow x = 3^2 = 9$

h. Rewrite each of the following using rational exponents and use inverses to solve for the variable. (You may need to use a calculator for some of them. Be careful!)

i. $\sqrt[5]{b} = 2$ ii. $\sqrt[5]{c^3} = 4.2$ iii. $\dfrac{1}{\sqrt[4]{d}} = \dfrac{1}{5}$

Let's look at some more problems that require the use of rational exponents.

4. Let's use a calculator to model bacteria growth. Begin with 25 bacteria.
   a. If the number of bacteria doubles each hour, how many bacteria are alive after 1 hour? 2 hours?
   b. Complete the chart below.

Time (hours)	0	1	2	3	4	5	6
Population	25	50					

   c. Write a function that represents the population of bacteria after *x* hours. (Check that your function gives you the same answers you determined above. Think about what if means if the base number is 1. What type of base number is needed if the population is increasing?)

d. Use this expression to find the number of bacteria present after 7 1/2 and 15 hours.

e. Suppose the initial population was 60 instead of 25. Write a function that represents the population of bacteria after x hours. Find the population after 7 1/2 hours and 15 hours.

f. Graph the functions in part (c) and (e). How are the graphs similar or different? What are the intercepts? What do the intercepts indicate? When are they increasing/decreasing? Are there asymptotes? If so, what are they? Do the graphs intersect? Where?

g. Revisit the graphs in problem (2). Compare with the graphs above. How are they similar and different? How do the equations indicate if the graphs will be increasing or decreasing?

h. Consider the following: Begin with 25 bacteria. The number of bacteria doubles every 4 hours. Write a function, using a rational exponent, for the number of bacteria present after x hours.

i. Rewrite the function in (g), using the properties of exponents, so that the exponent is an integer. What is the rate of growth of the bacteria each hour?

j. If there are originally 25 bacteria, at what rate are they growing if the population of the bacteria doubles in 5 hours? (Hint: Solve $50 = 25(1 + r)^5$.) What about if the population triples in 5 hours? (Write equations and solve.)

k. If there are originally 25 bacteria and the population doubles each hour, how long will it take the population to reach 100 bacteria? Solve the problem algebraically.

l. If there are originally 60 bacteria and the population doubles each hour, how long will it take the population to reach 100 bacteria? Explain how you solved the problem. (Solving the problem algebraically will be addressed later in the unit.)

THE POPULATION OF EXPONENTIA LEARNING TASK:

Upon landing on a nearly discovered planet, Exponentia, the astronauts discovered life on the planet. Scientists named the creatures Viètians (vee-et-ee-ans), after the French mathematician François Viète who led the way in developing our present system of notating exponents.

1. After observing the species for a number of years, NASA biologists determined that the population was growing by 10% each year.

In Math 2, you studied exponential functions. Use that knowledge to answer the following about the population of Exponentia.

a. If the estimated number of Viètians was 1 million in 1999 and their population is increasing 10% a year, write an equation for the population of Exponentia, P, as a function of the number of years, x, since 1999.

b. What was the population in 2005? What would it be in 2015 if the population growth rate remained the same?

c. Graph the function in part (a) on the interval $-15 \le t \le 15$.

i. What are the domain and range of the entire function?

Domain: ______________________ Range: _____________________

ii. Identify any asymptotes and intercepts. What do these characteristics tell us about the population on Exponentia from 1999 to 2009?

Asymptotes: _____________________ Intercepts: ______________________

Meanings: ____

iii. When is the function increasing / decreasing? What does this tell us about the population on Exponentia from 1999 to 2009?

d. Why would it be useful to have an equation that expresses the number of years since 1999 as a function of the population of Exponentia? That is, why might you want to be able to determine the year if you knew the population of Exponentia?

e. How can you use the graph in part (c) to determine the year in which the population reaches 2 million Viètians?

f. Write an equation, based on your function in part (a), for how you would find the year the population reaches 2 million. Use your graphing calculator to determine the solution.

We want to be able to solve the equation in part (f) algebraically. To solve $x^3 = 27$ for x, we apply the inverse of cubing, taking the cube root, to both sides of the equation. To solve $2x = 10$ for x, we apply the inverse of multiplying by x, dividing by 2, to both sides of the equation. Similarly, to solve the equation in (f), we need to apply the inverse of raising a value to the x power; we need to find the inverse of exponential functions.

After the following exploration, we will return to this problem and solve it algebraically.

2. **Graphs of exponential functions and their inverses.**

a. Graph the function $\boldsymbol{f(x) = a^x}$, where $a = ½, ¾, 2, 5$, and 3/2, on your graph paper. (You will have a total of 5 graphs.)

b. Using the above graphs, answer the following.

For $a > 0, a \neq 1$,

Domain: ________________ Range: __________________

Asymptote: ______________ Intercept: ________________

Increases when ________________; Decreases when ______________

c. Graph $f(x) = 2^x$. List at least two points that are on the graph of $f(x)$.

How is each a transformation of $f(x)$? For example, if $g(x) = 2^{x+4}$, it is a transformation of $f(x)$ 4 units to the left; $g(x) = f(x+4)$.

Explain which, if any, of the characteristics listed in part (b) will change.

Also, for each of the points you listed above, determine the location of those points after the transformations in the new equations.

(Think about how changes in the equations transform the graphs of the equations. If necessary, graph each function, along with $f(x)$, to answer this question.)

i. $g(x) = 2^{x-3}$
ii. $h(x) = 2^{2x}$
iii. $j(x) = 3(2^x)$
iv. $k(x) = 2^{-x}$
v. $l(x) = -2^x$
vi. $m(x) = 2^x+3$
vii. $n(x) = 2^x- 1$

d. We need to determine the inverse of an exponential function to algebraically complete problem 1 and to determine *x*-intercepts of more interesting exponential functions. List everything you know about graphs of inverse functions.

e. **Paper folding:** Using a piece of patty paper or wax paper, trace an accurate set of coordinate axes on the patty paper. Include all four quadrants.

   i. Draw an accurate graph of $f(x) = 2^x$ on the patty paper. Also draw the line $y = x$.

   ii. Fold the paper along the line $y = x$.

   iii. Trace the curve, as you can see it, on the outside of the paper.

   iv. Open the paper and trace the outside trace on the inside so that you have both the graph of the exponential function and the graph of its inverse on the front of the patty paper.

   v. Using the graph on your patty paper, make conjectures about the following characteristics of the inverse of the exponential function.

Domain: __________________ Range: _____________________

Asymptote: ________________ Intercept: ___________________

f. Using your lists from parts (b) and (e) and your conjectures in part (f), list the characteristics of the inverse of the exponential function. Draw the graph of this new function.

Domain: __________________ Range: _____________________

Asymptote: ________________ Intercept: ___________________

Graph:

3. A **logarithmic function** is the inverse function of an exponential function.

   a. In an exponential function, the input or independent variable is the exponent and the output or dependent variable is the value when the base is raised to an exponent. For example, in $f(x) = 3^x$, if $x = 4$, then $f(x) = 3^4 = 81$. What are the independent and dependent variables in a logarithmic function? (Explain in words, similar to what is done for exponents.)

Independent: ______________________________________

Dependent: ______________________________________

b. **Logarithmic notation:** To find the equation of inverses, we interchange *x* and *y* and solve for *y*.

$y = b^x \rightarrow x = b^y$

However, we need a method for solving for *y*. Just as when we need to take the inverse of a power function we use *nth* roots (square roots, cube roots, etc.), we need to introduce a new notation for the inverse of an exponential function.

For y and b both positive and $b \neq 1$,
$y = b^x$ if and only if $x = log_b\, y$

This is read as "*x* equals the log base *b* of *y*" or "*x* equals the base-*b* logarithm of *y*."

Note that the "base" is the "base" in both expressions. A logarithm of a positive number *y* is the exponent *x* you write *y* as a power of a base *b*.

Let's look at a few examples:

$10^2 = 100$ can be written as $log_{10}100 = 2$. Notice that the answer to the logarithm is the exponent from the first equation.

Evaluate $log_4 64$. This question asks for the value that when 4 is raised to that power is 64. In other words, 4 to what power equals 64? ________

Similarly, consider the following problem: $log_x 144 = 2$. Here, we consider the base that when raised to the second power, squared, yields 144. So $x^2 = 144$. Given that the base must be positive, what is the value of *x*? ______

c. **Practice with logarithms:**

   i. Write each exponential as a logarithm or logarithm as an exponential.
      1. $\log_{10}(1/100) = -2$
      2. $5^3 = 125$
      3. $\log_2 32 = 5$

   ii. Evaluate each logarithm.
      1. $\log_{10}(0.1)$
      2. $\log_3 81$
      3. $\log_2(1/16)$

4. $\log_9 81$

iii. Solve each logarithmic equation for *x*.
1. $\log_x 36 = 2$
2. $\log_4 4 = x$
3. $\log_7 1 = x$
4. $\log_8 x = 3$
5. $\log_5(3x + 1) = 2$
6. $\log_6(4x - 7) = 0$

d. **Understanding the logarithm definition:**
For y and b both positive and $b \neq 1$,
$y = b^x$ if and only if $x = \log_b y$

i. The statement of the relationship between exponents and logarithms states that $b \neq 1$. Explain why $b \neq 1$ if $y = b^x$ is an exponential function.

ii. The statement of the relationship between exponents and logarithms also states that *y* and *b* are both positive. Explain why the base, *b*, must be positive for $y = b^x$ to be an exponential function. Then explain why *y* must be positive if *b* is positive. Be sure to consider both positive and negative values of *x*.

e. **Common logarithms:** In logarithms, just as with exponential expressions, any positive number can be a base. However, the base of our decimal counting system, 10, is called the common base. Logarithms which use 10 for the base are called common logarithms and are notated simply as ***log x***. It is not necessary to write the base. The *log* button on your calculator computes common logarithms.

Solve the following: $\log 1000 = x$.
Understanding common logarithms can help solve more complex exponential equations.

Consider the following: Solve for *x*. $10^x = 250$

We know that $10^2 = 100$ and $10^3 = 1000$, so *x* should be between 2 and 3. Rewriting as a common logarithm can help solve the problem.

$\log 250 = x$ Using your calculator, find *x*. *x* = _______

f. Solve each of the following for *x* using common logarithms.
   i. $10^x = 15$
   ii. $10^x = 0.3458$
   iii. $3(10^x) = 2345$
   iv. $-2(10^x) = -6538$

4. **Solving exponential equations**: At numerous times in this unit, we have only been able to solve exponential equations using tables and graphs (e.g., How long does it take, #1f-g; How long does it take, #4l; Population of Exponentia, #1f). We can now use our understandings of common logarithms to solve these kinds of problems.[1]
   a. Consider the following problem: A bacteria culture in a Petri dish was begun with a single bacterium. Once the population reaches 100, the bacteria begin dying. How long does it take the population of the bacteria in the Petri dish to reach 100?

      Here's the equation we need to solve: $2^x = 100$

      We know that $2^6 = 64$ and $2^7 = 128$, so the answer must be between 6 and 7.

      We can rewrite each side of the equation as a power with a base of 10. That is, $10^a = 2$ and $10^b = 100$. So $a \approx 0.3010$ and $b = 2$.

      Explain how we know the values of *a* and *b*.

      Now, replacing 2 with $10^{.301}$ and 100 with 10^2, we have the following equation:

      $(10^{.301})^x = 10^2$
      $10^{.301x} = 10^2$ Power-to-power property
      $0.301x = 2$ Common base property
      $x \approx 6.6439$

      Explain why you might be confident that this is the correct answer.

   b. Solve each of the following equations, taken from earlier explorations, using the above method.
      i. $2 = 1.1^x$
         1. $10^a = 2$ and $10^b = 1.1$. Find *a* and *b*. a = _____ b = _____
         2. Use substitution to write an equation in terms of powers of 10.
            ____

[1] Adapted from *Discovering Advanced Algebra: An Investigative Approach* from Key Curriculum Press, 2004.

3. Use the power-to-power and common base properties. ____________________
4. Solve for *x*. x = ______________

ii. $300(.8)^x = 10$
iii. $100 = 60(2^x)$

c. This method can also be used to determine *x*-intercepts of exponential functions. Consider the equations in 2c of this task.
   i. Which functions would have *x*-intercepts if you were to graph them? How do you know without graphing them?

   ii. Select one of functions that *would* have an *x*-intercept. Determine, using the method above, the *x*-intercept.

   iii. Select one of functions that *would* have an *x*-intercept. Start solving for the *x*-intercept. How does your answer confirm that the graph of the function has no *x*-intercept?

5. **Graphs of logarithmic functions:** If $f(x) = b^x$, and the inverse of an exponential function with respect to x is a logarithmic function with respect to x. So $f^{-1}(x) = log_b x$.

In the paper folding activity above, you first graphed $f(x) = 2^x$ and then drew in the graph of the inverse function. What is the equation of $f^{-1}(x)$? _________________

We want to make explicit the characteristics that are common to ALL general logarithmic functions, regardless of the base. To do this, we need to look at graphs of different logarithmic functions.

a. Let's graph $y = log_5 x$ by hand.
   i. How can you use the fact that $y = log_5 x$ is the inverse of $y = 5^x$ to graph $y = log_5 x$?

   ii. Make a table of values to help you graph $y = log_5 x$. (Don't forget to use *x*-values between 0 and 1. Consider $y = 5^x$ in making your table.)

iii. Referring to your graph in 2f and your table above, draw the graph of $y = log_5 x$.

iv. Use the same procedure to draw the graph of $y = log_3 x$.

v. Compare the graphs of $y = log_5 x$ and $y = log_3 x$. Which one increases faster? How can you tell?

b. To better understand the characteristics of all logarithmic functions, use your the list feature on your graphing calculator or computer graphics tool to complete the following table.

	$y = log_5 x$	$y = log_3 x$	$y = log_{(1/3)} x$	$y = log_{(1/5)} x$
Domain				
Range				
Intercept				
Asymptote				
Zeros				
Increasing / Decreasing				

c. Using the information above, when do logarithmic functions increase? When do they decrease?

Give an example of a function that increases faster than the increasing functions in part (b). Explain how you determined your function.

Which of the above graphs decreased fastest?

Give an example of a function that would decrease faster than any of the ones above. Explain how you determined your function.

d. The definition of a logarithm states that x must be positive in $y = log_b x$. Accordingly, our domain is that x is positive. What happens when we transform the equation?

1. Consider $y = log\,(x - 3)$. This is a transformation of the logarithmic graph to the right 3 units. Sketch the graph of this function.

What is the new domain?

What is the new asymptote?

What is the new intercept?

How does the transformation relate to the changes above?

2. For more interesting functions like $y = log_4(3x – 5)$, we can use the same idea to determine graph attributes.

The domain will be determined by solving the inequality $3x – 5 > 0$. The asymptote will be determined by solving the equation $3x – 5 = 0$.

Find the domain. ___________ Find the asymptote. _______________

Generally, how do you find *x*-intercepts? Find the *x*-intercept for this function by solving the associated logarithmic equation..

3. Find the domain, asymptote, *x*-intercept, and *y*-intercept (if applicable) of $y = log_5(-3x + 8)$.

Domain: _____________ Asymptote: _______________

x-intercept: ___________ *y*-intercept: _______________

MODELING NATURAL PHENOMENA ON EARTH LEARNING TASK:

NASA has decided to send astronauts to explore Exponentia, a new planet recently discovered by NASA astronomers. Both the astronauts and the scientists realized that many preparations must be made before the trip. For one thing, the astronauts need to get their lives, including their financial situations, in order. And, in preparation for comparing the characteristics of Exponentia with natural phenomena on Earth, the scientists need to revisit, as two examples, how sound travels and how earthquakes are measured.

1. NASA must ensure that the astronauts have sufficient drinking water while they are traveling to Exponentia. Part of the scientists' jobs is to test the pH of the water that is to be stored on the space shuttle.

   pH is a measure of number of hydrogen ions, $[H^+]$, in a substance and measures how acidic or basic (alkaline) a solution is. When measuring pH, we often talk of measuring the acidity of a solution, assigning pH numbers between 0 and 14 that show relative acidity. A solution that has a pH less than 7 is acidic and a solution that has a pH greater than 7 is basic. A solution with a pH of 7 is neutral.

   $[H^+]$ is usually a very large or very small number. Because logarithms can be used to represent very large or very small numbers, we can use logarithms for this conversion. The pH is the negative logarithm of the concentration of free hydrogen ions, measured in moles per liter (moles/L). The conversion formula is $pH = -log\ [H^+]$.[2]

   a. Consider the general common logarithmic function, $f(x) = log\ x$. How will the graph of the pH conversion function, $g(x) = -\ log\ x$, differ from the graph of $f(x)$? Specifically, how, if at all, are the domain, range, intercepts, asymptotes, increasing/decreasing changed? What kind of graphical transformation is this?

   b. If water has a pH of 5, what is the concentration of hydrogen ions?

   $pH = -\ log\ [H^+]$ so we have $5 = -\ log\ [H^+]$. Rewrite this equation as an exponential equation. Then solve the equation.

   What if the water has a pH of 7? How does the concentration of hydrogen ions compare? Explain why your answers make sense.

   c. The normal $[H^+]$ range for drinking water is between approximately 10^{-6} and 3.16×10^{-9}. What is the approximate pH range?

[2] Information about acidity was gathered from a variety of web resources, including Wilkes University (www.water-research.net/ph.htm) and the Georgia Cooperative Extension Service (http://pubs.caes.uga.edu/caespubs/pubs/PDF/B1242-3.pdf).

d. The rover that was sent to test out the surface of Exponentia brought back a sample of a substance whose concentration of hydrogen ions measured 10^{-4}. Is this more or less acidic than our water? Explain. (Find the pH level of the substance.)

2. The explorer rover sent to Exponentia experienced planetary tremors similar to what we call earthquakes. The NASA scientists decided to train the astronauts to understand earthquake measurements in case they experienced these tremors while on Exponentia.

   a. In discussing the relative size of earthquakes, for each increase in one point on the Richter scale, the relative size increases ten-fold.[3]

   Complete the following table that shows the correspondence of relative size and Richter scale number.

Richter Scale Number	Relative Size
1	10
2	
3	
…	…
6	

   b. Write a logarithmic equation for the relationship between Richter number and relative size of the earthquake. Let relative size, *s*, be your independent variable and Richter scale number, *r*, be the dependent variable. What is the name for this special type of logarithm?

   c. The planetary rover experienced earthquakes of magnitudes 2 and 8. We want to know the relationship between the relative size of an earthquake that measures 8 on the Richter scale and one that measures 2.

[3] Some of the content for this question derives from the U.S. Geological Survey website (http://earthquake.usgs.gov).

What is the relative size of an earthquake of magnitude 2? Write this as a power of 10. ________

What is the relative size of an earthquake of magnitude 8? Write this as a power of 10. ________

What is the relationship between the relative sizes? (Hint: Subtract the exponents.)

d. In 2002, an earthquake of magnitude 7.9, one of the largest on U.S. land, occurred in the Denali National Park in Alaska. What was the relative size of this earthquake? (You will need to rewrite the logarithmic equation as an exponential equation.)

e. On April 29, 2003, an earthquake in Fort Payne, Alabama was felt by many residents of northern Georgia. The magnitude was 4.6. How does the relative size of the Alabama earthquake compare with the relative size of the Denali earthquake?

3. Rather than discuss relative size of an earthquake, we often prefer to discuss the amount of energy released by an earthquake. A formula that relates the number on a Richter scale to the energy of an earthquake is $r = 0.67 \log E - 7.6$, where r is the number on the Richter scale and E is the energy in ergs.

   a. Consider the general form of a logarithmic function, $f(x) = \log x$.
   How will the scale factor of 0.67, in the equation above, alter the graph of $f(x)$?

   What is the name of this transformation?

   How will the –7.6 alter the graph of $f(x)$?

   What is the name of this transformation?

   b. Determine the domain, range, intercepts, and asymptotes for the graph of $r = 0.67 \log E - 7.6$. Sketch the graph.

   Recall that for the domain and asymptote, we need to consider the value for which the logarithm is being taken. In this case, we are taking the logarithm of E.

   Domain: ________________ Range: _____________

   Asymptote: _______________ x-intercept: _____________

No option is provided above for a *y*-intercept. Why not? What type of transformations might result in the graphs having *y*-intercepts? (Investigate on your calculator if you would like.)

c. What is the Richter number of an earthquake that releases 3.9×10^{15} ergs of energy? (Be careful when inputting this into the calculator.)

$r = 0.67 \log E - 7.6$
$r = 0.67 \log (3.9 \times 10^{15}) - 7.6$
$r =$ ________

d. How much energy was released by the 2002 Denali earthquake? By the 2003 Alabama earthquake?

Denali earthquake: $r = 0.67 \log E - 7.6$
$7.9 = 0.67 \log E - 7.6$

Solve this equation for E by first solving for $\log E$ and then rewriting the equation in exponential form. $E =$ ____________

Alabama earthquake: $r = 0.67 \log E - 7.6$
$4.6 = 0.67 \log E - 7.6$
$E =$ ____________

4. Scientists use carbon-dating to determine the ages of carbon-based substances. The isotope Carbon-14 (C14) is widely used in radiocarbon dating. This form of carbon is formed when plants absorb atmospheric carbon dioxide into their organic material during photosynthesis. After plants die, no more C14 is formed and the C14 in the material declines exponentially.[4]

   a. Willard Libby was recognized with the Nobel Prize in Chemistry in 1960 for leading a team to develop a method of using carbon-14 dating to determine the age of carbon-based matter. Initially, circa 1949, Libby and associates believed the half-life of the C14 isotope, the amount of time it takes for half of the C14 to decay, to be approximately 5568±30 years. This is known as the **Libby half-life.**

[4] Credits for information on carbon-14 dating: www.c14dating.com/int.html

Using 5568 as the value of the half-life, find an equation that models the percentage of C14 in carbon-based material, such as a plant.

Let's assume that we start with a plant that contains 100% of its C14, at the moment it is cut.

If the initial amount is 100%, how much will remain at the end of one half-life? ______ In the equation $y = ab^x$, a = the initial amount. Will the b be greater than 1 or less than 1? ____ How do you know?

Using $y = ab^x$, substitute in the values you know, using the amount of time for the exponent. (The only unknown is the base, b.)

Now use a graphing calculator to find the value of b by allowing *Y1* to equal the left-hand side of your equation and *Y2* to equal the right-hand side of your equation. b = _________

Rewrite the equation $y = ab^x$ with the values for a and b. This is the equation for determining how long a plant has been decaying based on the amount of C14 remaining in the plant.

b. Later measurements led to the realization that the Libby half-life was too low. In actuality, the half-life of C14 is approximately 1.03 times the Libby half-life. This is called the **Cambridge half-life** and is the accepted value for the half-life of the C14 isotope.

Find the value of the Cambridge half-life. ___________

The actual value of the Cambridge half-life is 5730±40 years.

Will the base in the exponential equation that uses the Cambridge half-life be smaller or larger than the base in the equation above? Explain.

Using 5730 as the value of the half-life, find an equation that models the percentage of C14 in a plant. (Use the same procedure as above.)

c. How are the graphs of the functions in (a) and (b) similar? How are they different?

d. A plant contains 64.47% of its original carbon-14. According to this information, approximately how long ago did it die? (3627.6 years)

Using the equation from (b), substitute 64.47 for the ending amount, y.

We need to solve for x. Divide both sides by 100 and then use common logarithms to rewrite each side as 10 to a power and solve.

e. The planetary rover sent to Exponentia brought back a number of carbon-based plants. Assuming that the plants died the moment they were gathered by the rover in 2002, how much of the C14 would remain in 2222?

Use your equation in (b) to answer this question.

5. Before leaving for the exploration trip, one of the astronauts, Natalie Napier, met with her accountant to make sure that her money was earning as much as possible while she was gone.

a. In Math 2, you learned how to calculate investments that are compounded different numbers of time, as well as compounded continuously. State those formulas. Explicitly define the parameters (variables) in the formulas.

b. The compounded continuously formula employs the **transcendental number *e*.** Recall that a transcendental number is a number that is not the solution of a non-zero polynomial with rational coefficients. That means that *e* cannot be the solution of an equation such as $2x^2 + 3x - 4 = 0$. Can you think of another transcendental number? ______

Let's review how we can estimate the value of *e*. Using the table feature on your calculator (in the "ask" mode) and the compounded interest formula (not the compounded continuously formula), complete the following chart. Assume that you invest $1 at a 100% annual interest rate.

What is the equation, leaving only *n* and the ending amount as the unknowns?

____

Input this at *Y1*. Change the "Tblset" to start at 1 and set the independent variable option as "ask." Now go to the table and type the *n* values for which you want the dollar value.

Frequency of Compounding	**Number of times compounded in a year**	**Formula with values inputted**	**Dollar value at the end of 1 year**
Annual			
Semiannual			
Quarterly			
Monthly			
Weekly			
Daily			
Hourly			
Every Minute			
Every Second			

Now, use your calculator to find the value of *e*. ______________________

What do you notice when looking at the table and looking at the value for *e*? That is, how could we estimate the value of *e*?

c. Assume that Natalie has $10,000 to invest. Complete the following chart to show how much she would earn if her money was invested in each of the specified accounts for 10 years.

Frequency of Compounding	**Annual Interest Rate**	**Formula with Values Inputted**	**Amount After 10 Years**
1. Quarterly	3.65%		
2. Monthly	3.65%		
3. Daily	3.6%		
4. Continuously	3.6%		

Which account would you suggest for Natalie? ________________

d. Natalie is particularly interested in how long it will take her money to double.

Consider the first option. If her money doubles, she will have $20,000. So the equation we want to solve is

$20{,}000 = 10{,}000(1 + .0365/4)^{(4t)} \rightarrow 2 = 1(1 + .0365/4)^{(4t)} \rightarrow 2 = (1.009125)^{4t}$.

Remember that to solve some exponential equations, we must employ the inverse operation, logarithms. Recall that we can write each base as 10 to a power because a logarithm is the exponent when a base is raised to a power.

$2 = 10^{a} \rightarrow \log 2 = a \rightarrow a =$ ___________

$1.009125 = 10^{b} \rightarrow \log 1.009125 = b \rightarrow b =$ _____________

$2 = (1.009125)^{4t} \rightarrow 10^{\_\_\_} = 10^{\_\_\_ *4t} \rightarrow$ ______ = _____t $\rightarrow t =$ _______ *years*

Determine how long it will take Natalie's money to double under options 2 and 3.

Option 2:

Option 3:

e. We also want to determine how long it would take Natalie's money to double if it was compounded continuously.

So we have $20{,}000 = 10{,}000e^{.036t} \rightarrow 2 = e^{.036t}$.

Although we can employ the same method of writing each base as 10 to a power, taking advantage of common logarithms, whenever we have a base of *e*, we generally use **natural logarithms** instead. A natural logarithm is a logarithm with a base of *e*. Instead of writing $\log_e y = x \Leftrightarrow e^x = y$, we write $\ln y = x \Leftrightarrow e^x = y$.

Let's explore natural logarithms. First, compute 2 times ½. What do you get? What about 3/5 times 5/3? Why do we get the same answer?

Using your calculator, determine *ln e*. What do you get? Why?

So for our problem, we have $2 = e^{.036t}$. If we rewrite this equation as a logarithmic equation, we get $\log_e 2 = .036t$. We know that a logarithm with a base of *e* is a natural logarithm, so we can rewrite again as $\ln 2 = .036t$. Use your calculator to help you finish solving the equation for *t*. (Your answer should be close to the answers you got in part (d). If not, check that you are using your calculator correctly.)

f. Suppose Natalie invested her savings in an account that was compounded continuously at an annual interest rate of 3.7%. How long would it take her money to double?

First write the appropriate equation for this information. Then solve as in part (e).

Would your answer be different if she started with \$20,000? What about \$100,000? What about \$*x*? Explain. (Try some of the examples if you need to.)

g. The function $f(x) = e^x$ is a special case of the function $f(x) = a^x$.

i. With that in mind, sketch a graph of $f(x) = e^x$ and list the domain, range, intercepts, and asymptotes of $f(x) = e^x$.

ii. Sketch the graph of Natalie's earnings as a function of time, in years, if she invests \$1 into an account that is compounded continuously at an annual interest rate of 3%. (First, write the function. The graph the function.)

iii. How will the graph be different if Natalie invests \$10,000? Will the domain, range, intercept, and asymptote change? What, if anything, changes? What kind of transformation is this?

iv. Suppose that Natalie invests \$10,000. At the end of her mission, she receives a \$5,000 bonus. Write the equation for the amount of money that Natalie has after the mission.

How will the graph of the equation be different from the graph of the equation discussed in (iii)? What kind of transformation is this?

6. During practice flights before the mission, the astronauts needed to work on maintaining specific flying heights based on different amounts of air pressure.

A pressure altimeter is an instrument that finds the height of an airplane above sea level based on the pressure of the surrounding air. The height h and pressure p are related by the equation $h = -26{,}400 \ln (p/2120)$ where h is measured in feet and p is measured in pounds per square foot.[5]

a. The graph of $f(x) = \ln x$ is a special logarithm, as is $g(x) = \log_{10} x$. (Use your graphing utility as a tool.) Sketch the graphs of $f(x)$ and $g(x)$. State the domain, range, intercepts, and asymptotes of each.

b. Does $f(x)$ increase faster or slower than $g(x)$? What difference in the functions accounts for this difference? Give an example of a function that increases faster than both $f(x)$ and $g(x)$. Explain your reasoning.

c. Will the graph of the function stated in the problem be increasing/ decreasing? Explain.

d. The function $h = -26{,}400 \ln (p/2120)$ is a transformation of the graph of $f(x) = \ln x$. Identify each transformation and explain how each would alter the graph of $f(x) = \ln x$.

e. Consider the graph of $h = -26{,}400 \ln (p/2120)$.

If the intercept is usually (1, 0), what will the new intercept be? (Consider your transformations. Which transformation impacts the x-values of your graph?)

Based on the transformations, what should be the domain, range, and asymptote?

Use this to help you set your graphing window.

The control panel on the plane includes an instrument to measure height above sea level and air pressure. Suppose the air pressure instrument malfunctioned. Write an equation for determining the air pressure as a function of the height above sea level.

We want to solve the above equation for *p*. First, isolate the natural logarithm expression on one side of the equation.

Now use the fact that natural logarithms and natural exponentials, exponential functions with a base of *e*, are inverses to rewrite the expression without logarithms.

Solve for *p*.

f. If the plane's height is 2500 ft, what is the surrounding air pressure? (Use your equation in part (f).)